# Percents

Percents, as a topic, covers a lot of ground on the GMAT. It is all fundamentally based off the simple definition:

Given Percent x Whole = Desired Part

In other words, the "percent" of something describes a fraction of a total amount. That fraction, the "desired part" can be described as a "percent" relative to the total amount.

That seems unnecessarily complicated, of course, so I'd rather just use a few simple examples. Here's one to get started. We won't solve it right now, but just to give a small amount of context:

*What is 80% of 30?*

Notice for the moment how I'm writing all of these equations word-for-word *as math*, then converting them into fractions.

**Writing English as Math**

First, a little aside about writing English as math. It's vital and we don't need to get into exactly why–just yet, that's coming–but let's practice doing it this way. You'll thank me later.

The simple conversions I'm using are these:

*percent = /100*

*what = variable, generally x or y*

*is = equals sign (=)*

*of = multiply (x)*

As a simple example, let's try this one and then we'll work through a few more:

*What is 80% of 30?*

$$x = 80/100 * 30$$

See? Not difficult at all. I realize we haven't actually solved it yet, but that's coming.

**Some Examples of Percent Questions**

Let's start with the above.

**Example 1:**

*What is 80% of 30?*

$$x = 80/100 * 30$$

Next step, convert to fractions where possible. I know that 80/100 is the same as 4/5 so I will just sub in 4/5.

$$x = 4/5 * 30$$

Now you just need to cancel the 5 and the 30 to find 24, but you knew that already.

$$x = 4 * 6 = 30$$

**Example 2:**

*On a math test, John answered 85 percent of the questions correctly. If John answered 34 questions correctly, what was the total number of questions on the test?*

Here, one thing you might want to note at the beginning is that 85 and 34 are both multiples of 17. Just keep that in the back of your mind.

Write out the information you have and note what you're missing with a variable (*x* in this case). It might help to rephrase it slightly: *85 percent of x total questions is 34 questions.*

$$85/100 \ * \ x \ = \ 34$$

Here, what I would suggest you do is to consider *putting common factors together.* Remember how the 85 and the 34 share factors? This means I'd divide both sides by 85 to get this:

$$1/100 \ * \ x \ = \ 34/85$$

$$x/100 \ = \ 34/85$$

Remember how I mentioned *multiples of 17* above? This is where you use that knowledge to good effect:

$$x/100 \ = \ 34/85$$

$$x/100 \ = \ (2 \ * \ 17)/(5 \ * \ 17) \ = \ 2/5$$

Done.

**Example 3:**

***In basketball practice, Curtis attempted 65 shots, out of which 26 were baskets. What percent of her shots made baskets?***

Yes, Curtis can be a woman's name. Now that that's out of the way, let's note that 65 and 26 share a factor of 13.

Now rephrase the question in a useful way: *what percent of 65 shots is 26?*

Write it as math:

$$x/100 \ * \ 65 \ = \ 26$$

Put your common factors together:

$$x/100 \ = \ 26/65$$

Pull out those 13s:

$$x/100 \ = (2 \ * \ 13)/(5 \ * \ 13) = 2/5$$

And, if you're really at a loss for the answer in a GMAT Percents question, just assume it's 2/5. (Please don't actually do that).

**Percent Increase and Decrease**

Lots of books love to give two equations: one for percent increase and one for percent decrease.

The thing with percent increase and percent decrease is that they're exactly the same. It is literally the same process and same equation for both. There is no difference.

What happens is that if your answer is positive, it's an increase and if it's negative, it's a decrease. Surprise, surprise.

Let's take a look at an increase and a decrease and you'll start to see what I'm talking about…

$$Percent\ Change\ =\ \frac{final - initial}{initial} * 100$$

Another way you might conceptualize it is this:

$$Percent\ Change\ =\ \frac{change\ in\ value}{initial} * 100$$

That's it: you're just looking for the difference between the value you end up with (*final)* and the value you began with (*initial*), all relative to (dividing by, that is) the *initial*. Then it's all times 100.

**Two things:**

**First,** the "times 100" part is a bit of an afterthought: if you don't mind your answer as a fraction or decimal, you can forget it. In other words, if 0.63 = 63/100 = 63% all read the same to you, great.

They ought to, because they are all the same statement.

Honestly, I only worry about the *times 100* part when the answer choices include the *%* sign.

**Second,** why is it relative to the *initial?* Well, think about it like this: we want to know *what has changed*. This is only possible if we know where we started and make our measurement of *change* relative to that.

That is, if we did everything relative to the *final,* there would be no difference between the *final* and the *final* because they are the same value!

**Third,** as a bonus: if it wants a decrease it will usually say "decrease." If it wants an increase, it will usually say "increase." Crazy, right?

Let's look at a couple of examples:

*The stock market average last month went up from 8000 to 10000. What was the percent increase in the stock market last month.*

We have a formula. Let's try it:

$$Percent\ Change\ =\ \frac{final - initial}{initial} * 100$$

$$Percent\ Change\ =\ \frac{10000 - 8000}{8000} * 100$$

$$=\ \frac{2000}{8000} * 100\ =\ \frac{1}{4} * 100\ =\ 25\%$$

Let's look at another simple example:

*From 2010 to 2020, Albertville's population decreased from 78000 to 52000. What was the percent decrease in Albertville's population from 2010 to 2020?*

Notice that we're looking for a decrease here, so expect that we'll have a negative value. If we don't, we're in trouble…

$$Percent\ Change\ =\ \frac{final - initial}{initial} * 100$$

$$Percent\ Change\ =\ \frac{78000 - 52000}{78000} * 100$$

$$Percent\ Change\ =\ \frac{26000}{78000} * 100$$

Knock off the zeros, then notice the 13s!

$$Percent\ Change\ =\ \frac{-26}{78} * 100 = \frac{-2*13}{6*13} * 100 = \frac{-2}{6} * 100 = \frac{-1}{3}\ *\ 100\ =\ -\ 33.3\%$$

See, that wasn't difficult at all.

**Change When Given the Initial Amount**

Sometimes you'll be given the initial amount to work from and then be expected to find the *new amount after a change.*

This isn't quite the same as plugging into the above formula, so we'll have to consider how to arrange things slightly more carefully.

Let's look at an example:

*Standard price for a ticket is $40. On Tuesdays, the price is reduced by 30 percent. What is the price of a ticket on Tuesdays?*

There are a couple of different ways to approach this.

1) If I were to do it in my head, I'd simply calculate 30% of 40 = 3*10% of 40 = 3*4 = 12 and subtract that. That is,

$$40\ -\ 12\ =\ 28$$

2) The other way to do it would be to consider everything relative to 100%. Just think about the 100 and take off what you're discounting:

$$\frac{100-30}{100} * 40 = \frac{70}{100} * 40 = 0.7 * 40 = 28$$

   That's basically the same as saying 7/10*40, which just becomes 7*4 = 28. That might be easier if you're clear that 70% is 7/10 as a fraction.

**Change When Given the Final Amount and NOT the Initial**

The rule of thumb I use to distinguish between these types of questions is: "Am I working from 100%?" or "Am I given a value greater (or lesser) than 100% and asked to find 100%?" The latter is the case we're talking about here.

That's a situation more like this:

*This semester, student theater tickets cost $26, which is 30% more than last semester. What was the price for student theater tickets last semester?*

These questions limit our potential setup a bit more. We need to use a variable for the *initial* amount because, of course, we don't know it.

That is…

$$26 = (\frac{100+30}{100}) * x = 1.3x$$

Leaving us here…

$$26 = 1.3x \ ==> \ 26/1.3 \ = \ 20$$

Note how I ignored the decimal and basically just divided 26 by 13. You'll learn more about the *digits vs. decimals* shortcut in the *Arithmetic Shortcuts* guide.

**Combined Increases and Decreases**

It bears repeating: stages, stages, stages. Do your calculations in stages. Write them in tables, tables, tables to keep them straight.

*During the first part of a sale, the price Y of a sofa was reduced by 30 percent to price X. During the second part of the sale, the price X was* *increased by 10 percent to price Z. Final price Z is what percent of the initial price Y?*

Notice how the X, Y, and Z aren't in alphabetical order, which is a deliberate attempt to mislead. Oh, they'd never do that to you. Of course they wouldn't…

Let's start with the table:

| price | relative% |
|---|---|
| $Y$ | 100% of $Y$ |
| $X = Y - 0.3Y$<br>$= 0.7Y$ | ↓ 30% of $Y$ |
| $Z = X + 0.1X$<br>$= 1.1X$<br>$= 1.1(0.7Y)$<br>$= 0.77Y$ ← | ↑ 10% of $X$<br><br><br>final |

Notice how the table lets you take notes to keep track of what's going on? We can see pretty clearly what percent of which number we're talking about.

Just remember that when you apply a new increase or decrease that you're doing it relative to the place you are: for example, even though you start with *Y*, you're asked to do a 10% increase on *X*.

This will of course reflect on *Y,* but the actual increase is done on *X.*

Now all we need to do is to find the difference: we started with *Y* and are left with *0.77Y.*

This, of course, gives us:

$$0.77Y - Y = \; - 0.23Y =- 23/100 * Y \; = \; - 23\%$$

That is, a 23% decrease from the original value of *Y.*

**Adding a Flourish**

As a final brief note, you can always flip it around and apply the negative at the end because you're told it's a decrease.

To use this case as an example, $Y - 0.77Y = 0.23Y$ has the same absolute value as $0.77Y - Y = \; - 0.23Y$.

It's totally acceptable to do the first calculation and just add a negative at the very end given that it's a decrease. *Just remember to add it* (instead of wondering how you can reduce the value and have an increase–that'll just give you a headache).

Check out more advanced Percents in the *700+ Arithmetic Shortcuts Guide.*